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Scale of leptogenesis

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Abstract. We study the scale at which one can generate the lepton asymmetry of the universe which could then get converted to a baryon asymmetry during the electroweak phase transition. We consider the possibility that the Yukawa couplings could be arbitrarily small but sufficiently large to generate enough lepton asymmetry. This forbids the possibility of the $(B-L)$ breaking scale to be less than 10 TeV.

1 Introduction

In most grand unified theories (GUTs) the baryon asymmetry of the universe is generated during the GUT phase transition [1–3]. In these models the generated asymmetry also implies an equal amount of lepton asymmetry and hence there is no net $(B-L)$ asymmetry. On the other hand if the electroweak phase transition is a second order phase transition, then any primordial $(B+L)$ asymmetry generated during the GUT phase transition will be washed out [4] .

This situation can be saved in models where $(B-L)$ is broken at some intermediate scales. In this case a $(B-L)$ asymmetry can be generated through higgs decay or heavy Majorana neutrino decay if there is appropriate CP violation. The out-of-equilibrium condition then imposes a lower bound on this symmetry breaking scale to be around 10^7 GeV [2]. This bound is dependent on the fact that the Yukawa couplings are larger than 10^{-5}. Although esthetically this number sounds reasonable, nothing tells us definitely that the Yukawa couplings cannot be smaller than this. For example, if the Yukawa couplings relating the left handed leptons to the first generation right handed heavy neutrinos are of the order of 10^{-7}, then the out-of-equilibrium condition can be satisfied for even a TeV scale for left-right symmetry breaking. But the same Yukawa couplings enter in the expression for the generated $(B-L)$ asymmetry, which may then be very small.

In this article we study systematically the Boltzmann equations for the generation of the lepton asymmetry, and hence $(B-L)$ asymmetry, to find the lowest possible left-right symmetry breaking scale which satisfies the out-of-equilibrium condition and which generates enough baryon asymmetry after the electroweak phase transition. This scale, which is the only one in our model, is the the scale of the (B – L) symmetry breaking, the scale of the breaking of the $SU(2)_R$ symmetry breaking and is also the mass scale for the gauge bosons corresponding to these symmetries. This means that the left–right symmetric model of leptogenesis can be falsified experimently if the right handed charged gauge bosons corresponding to the $SU(2)_R$ symmetry are seen in the next generation experiments below the scale at which leptogenesis is possible. One would then have to look for some new scenario for generating a baryon asymmetry of the universe.

In the next section we briefly review the leptogenesis scenario, where one generates a lepton asymmetry when the right handed Majorana neutrinos decay. This then gets converted to a baryon asymmetry during the electroweak phase transition. Subsequently, we discuss the Boltzmann equations and the possible solutions for a low energy left-right symmetry breaking.

2 Model for leptogenesis

It was first proposed by Fukugita and Yanagida [5] that in extensions of the standard model, which include singlet heavy right handed neutrinos, it is possible to generate a lepton asymmetry at some intermediate scale ($\sim 10^{10}$ GeV), which can then get converted to a baryon asymmetry during the electroweak phase transition. The out-of-equilibrium condition is satisfied with small Yukawa couplings. In this scenario the heavy right handed neutrinos decay to light left handed neutrinos in out-of-equilibrium. The amount of asymmetry thus generated depends on the amount of CP violation [5,6].

In the following we shall consider a left-right symmetric extension [7] of the standard model, which incorpo-

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rates the right handed neutrinos and can explain the origin of parity violation in the standard model. In this extension there are new sources of CP violation which have a phenomenologically rich structure [8,9]. We consider the symmetry breaking chain, $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$ $[\equiv G_{LR}]$ $\overset{M_R}{\longrightarrow}$ $SU(3)_c \times SU(2)_L \times U(1)_Y$ $[\equiv G_{std}]$ $\overset{M_W}{\longrightarrow}$ $SU(3)_c \times U(1)_{em}$. The symmetry breaking $G_{LR} \rightarrow G_{std}$ takes place when the right handed triplet higgs field $\Delta_R \equiv$ (1,1,3,-2) acquires a vacuum expectation value (*vev*). In this model $(B-L)$ is a local symmetry. The breaking of the group G_{LR} also implies spontaneous breaking of $(B-L)$. Left-right parity implies the existence of another higgs field Δ_L which transforms as (1,3,1,-2) under G_{LR}. A higgs bi-doublet field ϕ (1,2,2,0) breaks the electroweak symmetry and gives masses to the fermions.

The fermion content of the model is, $q_{iL} \equiv [3,2,1,1/3]$, $q_{iR} \equiv [3,1,2,1/3]$, $\ell_{iL} \equiv [1,2,1,1]$ and $\ell_R \equiv [1,1,2,1]$, where $i = 1,2,3$ corresponds to three generations. The right handed neutrinos ($N_i \equiv \nu_{iR}$) are contained in ℓ_{iR} and we do not have to include them by hand. The Yukawa couplings in the leptonic sector are given by,

$$\mathcal{L}_{Yuk} = f_{ij}\overline{\ell_{iL}}\ell_{jR}\phi + f_{Lij}\overline{\ell_{iL}}^{c}\ell_{jL}\Delta_L^\dagger + f_{Rij}\overline{\ell_{iR}}^{c}\ell_{jR}\Delta_R^\dagger. \quad (1)$$

The scalar potential has many more terms compared to the standard model. We write down only those terms which contribute to the generation of the lepton asymmetry of the universe,

$$\mathcal{L}_{int} = g(\Delta_L^\dagger \Delta_R \phi\phi + \Delta_L \Delta_R^\dagger \phi\phi) + h.c.. \quad (2)$$

The *vev*s of these fields are not independent. We consider the minimum of the complete potential which satisfies $v_L \ll v_R$ and $v_L \approx v^2/v_R$, where $v_{L,R}$ and v are the *vev*s of the fields $\Delta_{L,R}$ and ϕ respectively. We also assume that the left-right parity ($D-$ parity) is not broken, and hence the masses of the fields Δ_L and Δ_R remain the same even after the breaking of G_{LR}, *i.e.*, $m_{\Delta_L} = m_{\Delta_R} = m_\Delta \approx v_R$. At this scale v_R, the $(B-L)$ local symmetry is also broken by two units, which gives rise to Majorana masses of the neutrinos and neutron-antineutron oscillations.

The $\Delta_{L,R}$ can now decay into two leptons, while $\Delta_{L,R}^\dagger$ decay into two antileptons:

$$\Delta_{L,R} \rightarrow \ell_{L,R} + \ell_{L,R}. \quad (3)$$

$$\Delta_{L,R}^\dagger \rightarrow \ell_{L,R}^c + \ell_{L,R}^c. \quad (4)$$

These interactions, along with the scalar interactions

$$\Delta_{L,R} \rightarrow \phi + \phi, \qquad \rightarrow \phi^\dagger + \phi^\dagger,$$

give rise to lepton number violation. The interference of the tree level diagram and the one loop diagram of Fig. 1 can then give rise to a lepton asymmetry in the decay modes of (3) and (4) for the left-handed triplets Δ_L given by,

$$\epsilon_\Delta \approx \frac{1}{8\pi|f_{Lij}|^2}\mathrm{Im}[g^* f_{Lij}^* f_{ik} f_{jk}] F\left(\frac{g^*}{f_{Rkk}}\right), \quad (5)$$

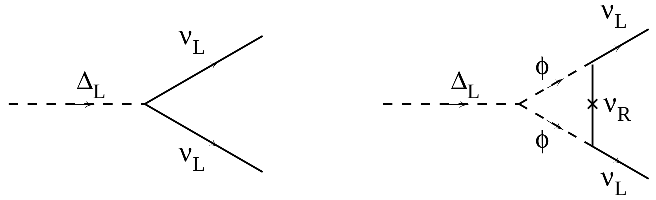

Fig. 1. Tree and one loop diagrams of lepton number violating triplet higgs Δ_L decay

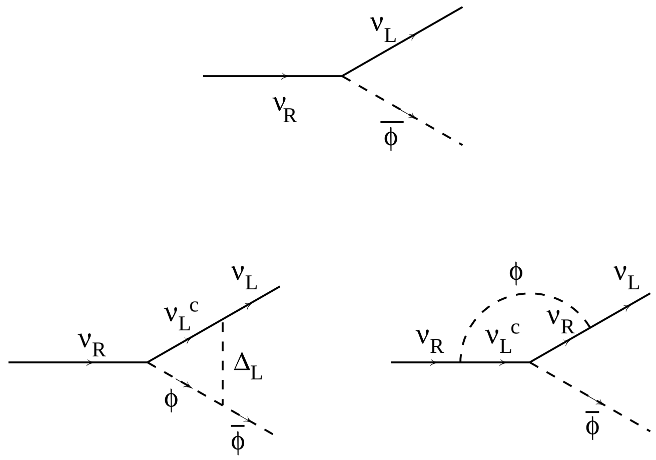

Fig. 2. Tree and vertex correction type one loop diagrams contributing to the generation of lepton asymmetry

where $F(q) = \ln(1 + 1/q^2)$. The quantity $[g^* f_{Lij}^* f_{ik} f_{jk}]$ contains a CP violating phase and so can be complex.

The fields Δ_R cannot generate any asymmetry because the Δ_R can decay through lepton–number violating decay mode of two higgs only after the Δ_L has acquired a *vev*. But Δ_L acquires a *vev* at a scale less than a few eV due to a seesaw suppression and gives a coupling for the Δ_R to decay which is highly suppressed. Although the left handed higgs triplets Δ_L can generate lepton asymmetry through their lepton number violating interactions, the scattering process $\Delta_L + \Delta_L \rightarrow W_L + W_L$ should make their number density the same as the equilibrium density. To avoid this possibility we find there is a strong constraint on the mass of the triplet higgs, namely, $M_\Delta > 10^{14}$ GeV [10]. At temperatures below this scale the density of the Δ_L always is the equilibrium density and hence cannot generate a lepton asymmetry of the universe. Fortunately the Δ_L are heavier than the right handed neutrinos. Thus for the generation of lepton asymmetry at low energy the main contribution comes from the decays of heavy right handed neutrinos.

The *vev* of Δ_R spontaneously gives a Majorana mass to the right handed neutrinos. This in turn allows the decay of ν_R into a lepton and an antilepton,

$$N_i \rightarrow \ell_{jL} + \bar{\phi}, \quad (6)$$

$$\rightarrow \ell_{jL}{}^c + \phi. \quad (7)$$

In the case of decays of the right-handed neutrinos there are two types of loop diagrams which can interfere with the tree level decays of (6) and (7) which are shown in

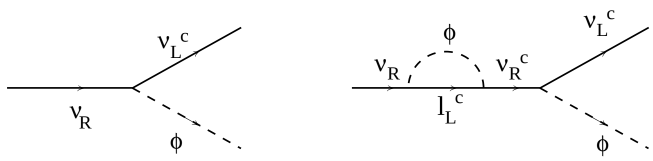

Fig. 3. Tree and self energy correction type one loop diagrams contributing to the generation of lepton asymmetry

Fig. 2. The interference of the tree level diagram and the one loop diagrams of Fig. 2 generates a lepton asymmetry given by,

$$\epsilon_\nu \approx \frac{1}{4\pi|f_{ik}|^2}\mathrm{Im}[f_{ik}f_{il}f^*_{jk}f^*_{jl}]\frac{f_{Rii}}{f_{Rkk}}. \tag{8}$$

In addition to these one loop vertex type corrections, there are self energy type corrections which also contribute to the lepton asymmetry of the universe (Fig. 3). In the limit of large mass–squared difference between the two generations of heavy neutrinos, various aspects of this interference of the tree and the one-loop diagrams (Fig. 3) have been discussed and calculated in the literature [11–16].

There were attempts to use the oscillation phenomenon to generate a baryon asymmetry [11]. For Majorana neutrinos the self-energy diagrams were considered in ref [12], but it was not realized that this corresponds to CP violation of the indirect type [13]. This realization created renewed interest in the field and an understanding of the phenomenon became important. It has now been clarified [14] that although the unitarity condition seems to imply that it is not possible to generate a lepton asymmetry through the oscillation process, if the system departs from equilibrium and the real intermediate states are properly taken into account, then CP violation can occur.

It was shown in [13] that in the indirect CP violation there is a resonance phenomenon when the two heavy neutrinos are almost degenerate. This has been confirmed by other calculations [16]. However, the different calculations give somewhat different results. Here we will take a form which gives the largest effect [13], one which will give the smallest lower bound on the left-right symmetry breaking scale.

In the case of a small mass difference, but ignoring the width of the Majoranas, which is, of course, necessary when the mass difference becomes very small, δ is given by [13]:

$$\delta = 2\,\pi\, g^{ab}\mathcal{C}\frac{M_1M_2}{M_2^2-M_1^2} \tag{9}$$

where

$$\mathcal{C} = -\frac{1}{\pi}\mathrm{Im}[\sum_\alpha(f^*_{\alpha 1}f_{\alpha 2})\sum_\beta f^*_{\beta 1}f_{\beta 2})] \\ \times\left(\frac{1}{\sum_\alpha|f_{\alpha 1}|^2}+\frac{1}{\sum_\alpha|f_{\alpha 2}|^2}\right) \tag{10}$$

As mentioned above, this contribution becomes significant when the two mass eigenvalues are close to each other. For very large values of the mass difference the two contributions ϵ_ν and δ are of the same order of magnitude.

We now have to consider only the Δ_L, Δ_R and $N_{1,2}$ decay processes. We assume N_1 to be the lightest of the right handed neutrinos. If the masses of N_1 and N_2 are almost degenerate, their decay widths can become larger than the mass difference. In this case both the neutrino decays will contribute to the lepton asymmetry of the universe. The decay widths for $\Delta_{L,R}$ and N_1 are,

$$\Gamma_{\Delta_{L,R}} = \frac{|f_{[L,R]ij}|^2}{16\pi}M_\Delta \qquad \text{and}$$
$$\Gamma_{N_i} = \frac{|f_{1j}|^2}{16\pi}M_N, \tag{11}$$

where M_N is the mass of N_1. Since we assumed $M_N < M_\Delta$, at low energy the Δ_L decay will erase all lepton asymmetry and then the N_1 decay will generate the required asymmetry. For this reason while working the details of the Boltzmann equation we take the effect of only $\epsilon = \epsilon_\nu + \delta$. We shall now proceed to solve the Boltzmann equations including all these contributions and the scattering processes.

3 Solutions of the Boltzmann equations

The evolution of lepton and neutrino densities is governed by the Boltzmann equations. We start by deriving the Boltzmann equation for the neutrino number density n_i. The equation governing the evolution of n_i is:[2,10]

$$\begin{aligned}\dot{n_i} + 3Hn_i &= \int\mathrm{d}\Pi_i\mathrm{d}\Pi_1\mathrm{d}\Pi_2\ (2\pi)^4\delta^{(4)}(p_i-p_1-p_2)\\ &\quad\times\Big\{-f_i(p_i)|\mathcal{M}_0|^2\\ &\quad+\frac{1}{2}(1+\epsilon)|\mathcal{M}_0|^2f_l(p_1)f_{\Phi^c}(p_2)\\ &\quad+\frac{1}{2}(1-\epsilon)|\mathcal{M}_0|^2f_{\bar{l}}(p_1)f_\Phi(p_2)\Big\}\\ &= \int\mathrm{d}\Pi_i\mathrm{d}\Pi_1\mathrm{d}\Pi_2\ (2\pi)^4\delta^{(4)}(p_i-p_1-p_2)\\ &\quad\{-f_i(p_i)+f_i^{eq}(p_i)\}\,|\mathcal{M}_0|^2+\mathcal{O}(\epsilon,\mu/T)\\ &= -\Gamma_i(n_i-n_i^{eq}),\end{aligned} \tag{12}$$

where n_i^{eq} is the equilibrium number density of the N_i, and Γ_i is the thermally averaged decay width of N_i. The term on the left-hand side is the time derivative of n_i, plus a term which accounts for the dilution effect of the expansion of the universe. The integration is over phase space $\mathrm{d}\Pi$ and the phase space densities f, are given by the Maxwell-Boltzmann statistics:

$$f_i(E) = \exp[-\frac{(E-\mu_i)}{T}],\quad f_l(E) = \exp[-\frac{(E-\mu)}{T}],$$
$$f_\Phi(E) = \exp[-\frac{(E+\mu)}{T}], \tag{13}$$

where μ is the chemical potential. The matrix element $\mathcal{M}_0$ is defined by:

$$|\mathcal{M}(N \to \bar{l}\Phi)|^2 = |\mathcal{M}(l\Phi^c \to N)|^2 = \frac{1}{2}(1+\epsilon)|\mathcal{M}_0|^2,$$
$$|\mathcal{M}(N \to l\Phi^c)|^2 = |\mathcal{M}(\bar{l}\Phi \to N)|^2 = \frac{1}{2}(1-\epsilon)|\mathcal{M}_0|^2, \quad (14)$$

where ϵ is a measure of CP-violation. The interference of the tree and the one loop graphs give the $CP-$violating contribution to the Boltzmann equation. This is what allows the lepton asymmetry to grow:

$$\left[|\mathcal{M}(N \to l\Phi^c)|^2 - |\mathcal{M}(N \to \bar{l}\Phi)|^2\right] = \epsilon|\mathcal{M}_0|^2.$$

It is more convenient to work with the variables:

$$Y_i = n_i/s, \;\; x = M_i/T = [2H(x=1)t]^{\frac{1}{2}}, \quad (15)$$

where M_i is the mass of N_i; $s = g_* n_\gamma$ is the entropy density of the universe; g_* is the total spin degrees of freedom; n_γ is the equilibrium photon density of the universe and Y_i the number of neutrinos per co-moving volume element. $H = 1.7\sqrt{g_*}T^2/M_P$ is the Hubble constant, M_P is the Planck mass, taken to be 10^{18} GeV.

Thus:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}x} = -K\gamma x(Y_i - Y_i^{eq}), \quad (16)$$

where we have used:

$$n_\gamma = s/g_*, \;\; \frac{\mathrm{d}s}{\mathrm{d}t} = -3s\frac{\dot{R}}{R} = -3sH, \quad (17)$$

and:

$$K = \frac{\Gamma_i(x=1)}{H(x=1)}, \;\; \gamma = \frac{\Gamma_i(x)}{\Gamma_i(x=1)}, \quad (18)$$

with:

$$Y_i^{eq} = n_i^{eq}/s = \begin{cases} g_*^{-1} & x \ll 1 \\ g_*^{-1}\sqrt{\pi/2}x^{3/2}\exp(-x) & x \gg 1 \end{cases}.$$

In solving the Boltzmann equations, we make the further change of variables, $X = g_* Y$, thus:

$$\frac{\mathrm{d}X_i}{\mathrm{d}x} = -K\gamma x(X_i - X_i^{eq}). \quad (19)$$

This is the Boltzmann equation for the evolution of neutrino number density, with the initial condition $X_i(0) = 1$.

We now derive the Boltzmann equation for $L = \frac{1}{2}(l - \bar{l})$, where we have to take into account the processes $l + \Phi^c \leftrightarrow \bar{l} + \Phi$ mediated by a right-handed Majorana neutrino, as well as the processes:

$$\bar{l} \leftrightarrow N\Phi^c. \quad (20)$$

The Boltzmann equation for the number density of the light left-handed leptons is:

$$\begin{aligned} \dot{n}_l + 3Hn_l = &\int \mathrm{d}\Pi_N\, \mathrm{d}\Pi_1\, \mathrm{d}\Pi_2\, (2\pi)^4\, \delta^{(4)}(p_N - p_1 - p_2) \\ &\times [-(1-\epsilon)f_l(p_1)f_{\Phi^c}(p_2) \\ &+(1+\epsilon)f_N(p_N)]\,|\mathcal{M}_0|^2 \\ &+2\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2\,\mathrm{d}\Pi_3\,\mathrm{d}\Pi_4\,(2\pi)^4 \\ &\delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\times \left[-f_l(p_1)f_{\Phi^c}(p_2)|\mathcal{M}'(l\Phi^c \to \bar{l}\Phi)|^2\right. \\ &\left.+f_{\bar{l}}(p_3)f_\Phi(p_4)|\mathcal{M}'(\bar{l}\Phi \to l\Phi^c)|^2\right] \\ &+\mathcal{O}(\epsilon n_l + n_l^2). \end{aligned} \quad (21)$$

The origin of the various terms is described below.

The interaction term comes from CP-violation in the decay and inverse decay of the heavy Majorana neutrinos and is proportional to ϵ, the CP-violating phase and the squares of the matrix element $|\mathcal{M}_0|$. The second term describes the $2 \leftrightarrow 2$ lepton number violating scattering process. Since the decays of the real intermediate states and their inverse decays have already been included in the CP violating contribution, they have been subtracted from the scattering term mentioned above.

The corresponding equation for $\bar{l}$ is obtained as usual, by interchanging $l \leftrightarrow \bar{l}$, $\epsilon \leftrightarrow -\epsilon$, etc. To obtain the Boltzmann equation for the evolution of $n_L = \frac{1}{2}(n_l - n_{\bar{l}})$ we subtract the equation for $n_{\bar{l}}$ from that for n_l and multiply by a factor of 1/2:

$$\begin{aligned} \dot{n}_L + 3Hn_L = &\epsilon\Gamma_i(n_i - n_i^{eq}) - n_L(n_i^{eq}/n_\gamma)\Gamma_i \\ &-2n_L n_\gamma < \sigma|v| >, \end{aligned} \quad (22)$$

$$\Gamma_j = \frac{h_{\alpha j}^2}{16\pi}. \quad (23)$$

where $|\mathcal{M}'(l\Phi^c \to \bar{l}\Phi)|^2$ and $|\mathcal{M}'(\bar{l}\Phi \to l\Phi^c)|^2$ are the squares of the matrix elements for $2 \leftrightarrow 2$ L-nonconserving scatterings with the part due to real, intermediate-state N's removed and Γ_j is the decay rate of the right handed neutrino N_j. Here the quantity:

$$\begin{aligned} < \sigma|v| > = &\int \mathrm{d}\Pi_1\mathrm{d}\Pi_2\mathrm{d}\Pi_3\mathrm{d}\Pi_4(2\pi)^4 \\ &\delta^{(4)}(p_1 + p_2 - p_3 - p_4)f_l(p_1)f_l(p_2) \\ &|\mathcal{M}'(l\Phi^c \to \bar{l}\Phi)|^2/n_\gamma^2, \end{aligned} \quad (24)$$

is the velocity-averaged $2 \leftrightarrow 2$ L-violating cross-section. The presence of the term $-\epsilon\Gamma_i n_i^{eq}$ is due to the CP-violating part of $|\mathcal{M}'(l\Phi^c \to \bar{l}\Phi)|^2 - |\mathcal{M}'(\bar{l}\Phi \to l\Phi^c)|^2$.

In parallel to the calculation for the n_i, we obtain:

$$\frac{\mathrm{d}Y_L}{\mathrm{d}x} = \epsilon K\gamma x(Y_i - Y^{eq}) - g_* Y^{eq} Y_L K\gamma x - \frac{2Y_L\Gamma_s x}{H(x=1)}, \quad (25)$$

where $\Gamma_s = n_\gamma < \sigma|v| >$, with the initial condition $Y_L(0) = 0$.

To solve the set of coupled differential equations, we take for Γ_i, γ and Γ_s:

$$\Gamma_i = \frac{f^2}{16\pi}M_i \begin{cases} x & x \ll 1 \\ 1 & x \gg 1 \end{cases}, \quad (26)$$

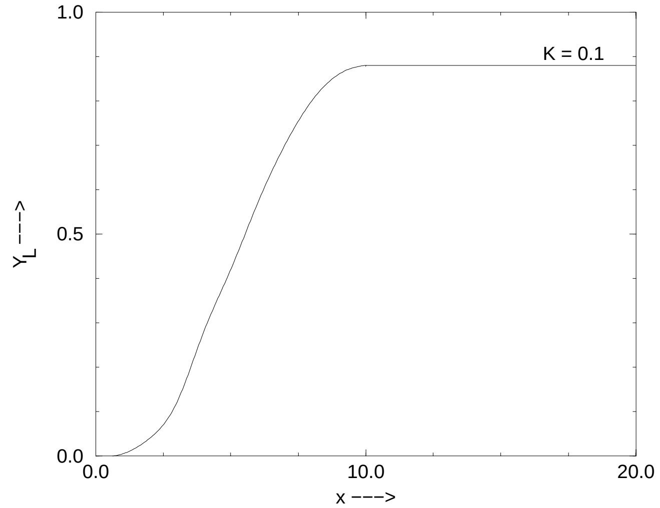

Fig. 4. Lepton asymmetry Y_L grows steadily to a constant asymptotic value ϵ for $K < 1$

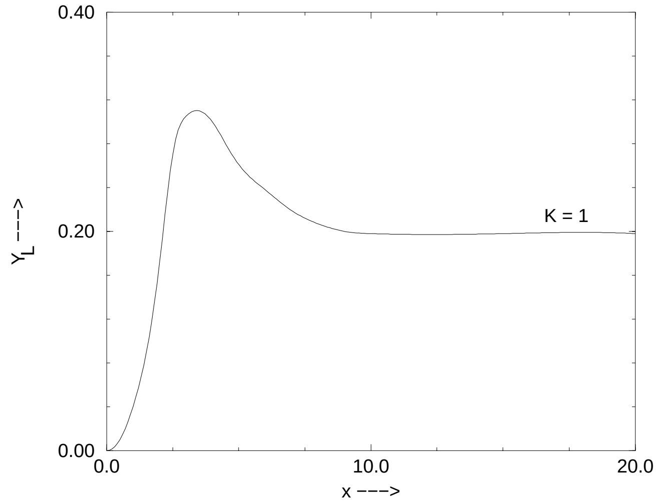

Fig. 5. For $K = 1$, lepton asymmetry starts depleting before reaching a constant value. The asymptotic constant value is thus much less than ϵ

$$\gamma = \begin{cases} x & x \ll 1 \\ 1 & x \gg 1 \end{cases}, \tag{27}$$

and:

$$\Gamma_s = \frac{3f^4}{4\pi}\frac{M_i}{x}\begin{cases} 1 & x \ll 1 \\ \frac{2}{3}\frac{1}{x^2} & x \gg 1 \end{cases}. \tag{28}$$

Note that for small x the scattering term is proportional to $1/x$ whereas the decay rate behaves like x. The functions γ, Γ_i, and Γ_s must be specified in the region $x \approx 1$, in order to find a solution for the differential equations. We take:

$$\gamma = 1 - \exp(-x), \tag{29}$$

$$\Gamma_i = \frac{M_i f^2}{16\pi}\left(1 - \exp(-x)\right), \tag{30}$$

and:

$$\Gamma_s = \frac{3f^4 M_i}{4\pi x}\left(1 - \exp[-2/(3x^2)]\right), \tag{31}$$

which are a good approximation to the functions (26), (27) and (28) above, in the regions $x \ll 1$ and $x \gg 1$. The numerical solutions to the Boltzmann equations are then obtained for different values of f, K and ϵ.

We summarize our observations below and in Figs. 4–7. We want to find the lowest right–handed symmetry breaking scale which can give leptogenesis. If a gauge boson corresponding to the $SU(2)_R$ symmetry is seen at energies lower than this lowest allowed scale then the possibility of leptogenesis is ruled out. One would have to look alternative ways to generate a baryon asymmetry of the universe. From our analysis of the Boltzmann equation we can only get a bound on the lightest right handed neutrino (N_1). However, the masses of the right handed neutrinos are related to the left-right symmetry breaking scale by,

$$M_1 = f_{R11} < \Delta_R >= f_{R11} M_R.$$

Considering the fact that the Yukawa couplings are smaller than 1, we can translate a bound on M_1 to a bound on M_R. In other words, when we say, M_R is greater than

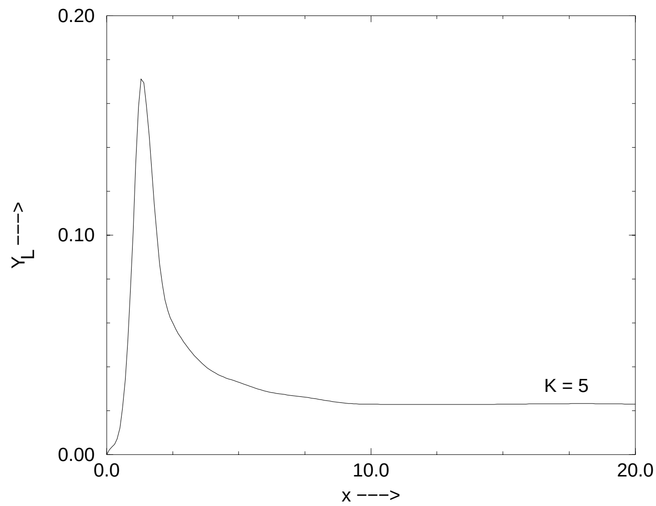

Fig. 6. For large $K > 1$ the behaviour is similar to $K = 1$. For $K = 5$ the asymptotic value is further depleted

some value, it actually means that M_1 is greater than that value and M_R is much heavier.

To demonstrate our results we presented only a few representative graphs, with the coupling constants being taken to be of the order of 1. However, in our analysis we have considered much smaller coupling constants also. The figures depend on the value of K and the relative magnitudes of the scattering and the decay rates for a specific choice of the coupling constant. As we change the coupling constants, the values at which the scattering process becomes dominant compared to the CP violating decay rates will change, but the nature of the graphs will remain the same.

For $K \ll 1$, the amount of lepton asymmetry grows cubically to a constant asymptotic value which we call Y_L^{asym} ($K \ll 1$) as shown in Fig. 4. In all these figures we have taken the values of the couplings to be of the order of 1. The nature of the curve is independent of the choice

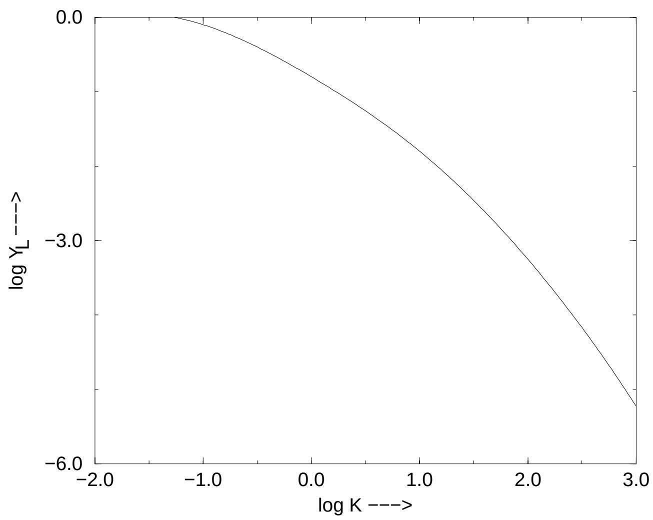

Fig. 7. The asymptotic value of the lepton asymmetry for different values of K in $log_{10} - log_{10}$ graph. For $K = 1000$ the lepton asymmetry drops to 8×10^{-6}

of the Yukawa couplings. This asymptotic value is given by, $Y_L(asym) = (\epsilon + \delta)/g_*$. This is the case when the decay rate of N_1 is less than the expansion rate of the universe. In this case the scattering rates are also less than the expansion rate of the universe. If the mass difference between N_1 and N_2 is of the same order of magnitude to their masses, the contributions of ϵ and δ become comparable. This case has been discussed extensively in the literature and the constraint on the scale of $(B-L)$ breaking obtained from this condition is $M_R > 10^7$ GeV with the Yukawa couplings to be of the order of 10^{-5}. Somewhat smaller values of the Yukawa couplings can reduce the scale of $(B-L)$ breaking to a lower value. However the amount of lepton asymmetry will be inadequate unless there is a large hierarchy among the Yukawa couplings of different generations.

When the mass difference $|M_1 - M_2| \sim 10^{-3} M_{1,2}$, δ can be three to four orders of magnitude larger than ϵ. In this case we can consider $|f| \sim 10^{-7}$ and still get adequate amount of lepton asymmetry. This will then allow us somewhat smaller right handed symmetry breaking scale $M_R > 10^5$ GeV.

When K is of the order of unity or more, the lepton asymmetry vanishes before $T = M_1$. From $T = M_1$ onwards the lepton asymmetry starts increasing from its initial value of $Y_L = 0$, but as it approaches the asymptotic value of $Y_L^{asym}(K \ll 1)$ the scattering processes becomes important and start depleting it exponentially (as shown in Figs. 5 and 6). Unlike the common folklore that when the system is in equilibrium the asymmetry falls exponentially fast, here the scattering processes become unimportant and the lepton asymmetry reaches its new asymptotic value, which is less than $(\epsilon + \delta)/g_*$). For $K = 1$ the suppression factor is about 1/5 and for $K = 5$ it is .02. Figure 7 shows the fall of this asymptotic values of the lepton asymmetry for different values of K. For $K = 1000$ the suppression factor is about 8×10^{-6}. In grand unified theories where the heavy gauge bosons decay generates an $(B+L)$ asymmetry it was shown [2] that this suppression factor is $\approx [K(\ln K)^{0.6}]^{-1}$. But in the case of leptogenesis, as Fig. 7 shows, this suppression is much faster than linear. For large mass difference $|M_1 - M_2|$, Y_L is approximately proportional to f^2 when all the f's are of the similar order of magnitude and $K \sim 10^{-3} f^2 M_P/M_1$. Since K and Y_L both are proportional to $|f|^2$, we cannot improve the bound on the right handed symmetry breaking scale with large K. However, since the suppression for large K is almost quadratic we can at most lower the scale of left-right symmetry breaking by one order of magnitude to about 10^4 GeV.

For very large K, if $|f|^4 \geq 10^{-15} \frac{M_1}{(100 \text{ GeV})}$, the scattering processes become larger than the expansion rate of the universe. In this case the lepton asymmetry decreases exponentially and never reaches any asymptotic value. In this region the actual equilibrium condition is satisfied.

4 Summary and conclusion

We have studied the possible scale of left-right symmetry breaking which can generate enough lepton asymmetry of the universe. For simplicity we assumed that the order of magnitude of the Yukawa couplings of the heavy neutrinos are similar. Since there is the freedom to choose the Yukawa couplings by several orders of magnitude, one resorts to naturalness to constraint the scale of leptogenesis. In this article we show that even including the resonance condition, which can enhance the amount of CP violation by few orders of magnitude for almost degenerate heavy neutrinos, the lowest scale for leptogenesis can not be less than 10^4 GeV. We have also considered the situation when the interaction rates are slightly larger than the expansion rate of the universe, when it is still possible to generate a lepton asymmetry of the universe. However, even in this case the lower limit on the scale of left-right symmetry breaking remains the same. As a result, if a right handed charged gauge boson is observed in one of the next generation accelerators we have to resort to new scenarios to generate a baryon asymmetry of the universe.

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